ECE 627 Spring 2008

FINAL EXAMINATION
June 9, 2008
Open book

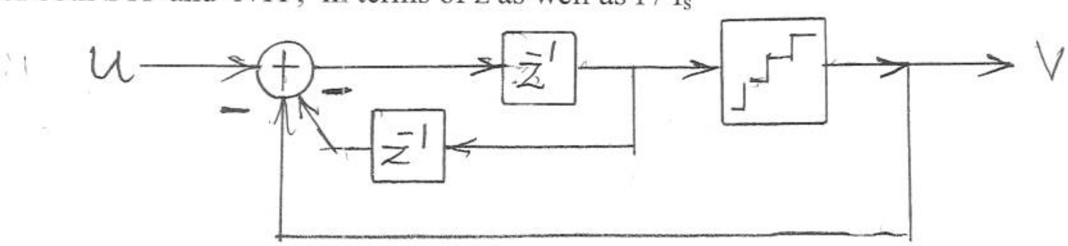
Name: ______________________________

1. The signal transfer function (STF) of a delta-sigma modulator is 1, the noise transfer function (NTF) is $(1 - z^{-1})^2$. The LSB voltage of the quantizer is 0.1 V. The input signal is $u(t) = \sin(2\pi ft)$, where $f = 1$ kHz. The clock frequency is 128 kHz.

a. Find the largest possible difference between u(n) and v(n).

b. Find the largest change |y(n) - y(n-1)| in the input signal of the quantizer.

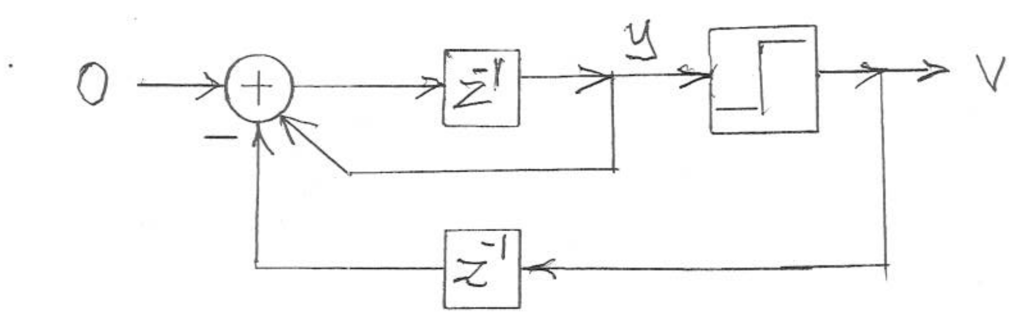
2. Find STF(z) and NTF(z) for the modulator shown below. Calculate the zeros and poles of both STF and NTF, in terms of z as well as f / f_s



3. The modulator shown below has zero input at all times. The quantizer input y(n) is zero for n = -1 and n = -2. The quantizer characteristics is

$$v(n) = -1 \text{ if } y(n) < 0; \qquad v(n) = +1 \text{ if } y(n) \geq 0 .$$

Calculate and plot v(n) for n = 0, 1, 2, ..., 10. (Hint: it is periodic.)



1. a. $u(n) = \sin(2\pi f n/f_s) = \sin(n\pi/64)$

$$V(z) = U(z) + (1 - 2z^{-1} + z^{-2})E(z)$$

$$v(n) = u(n) + e(n) - 2e(n-1) + e(n-2)$$

$$|v(n) - u(n)|_{max} = 4|e(n)|_{max} = 2V_{LSB}$$

$$\underline{|v - u| \leq 0.2\,V}$$

b. $y(n) = v(n) - e(n) = u(n) - 2e(n-1) + e(n-2)$

$$y(n) - y(n-1) = u(n) - u(n-1) - 2e(n-1) + 3e(n-2) - e(n-3)$$

$$\underline{|y(n) - y(n-1)|_{max}} = \sin(\pi/64) + 6|e_n|_{max}$$

$$\cong 0.0491 + 0.3 = \underline{0.3491\,V}$$

2. $V = E + z^{-1}[U - V - z^{-1}(V - E)]$

$$(1 + z^{-1} + z^{-2})V = z^{-1}U + (1 + z^{-2})E$$

$$STF = \frac{z}{z^2 + z + 1}, \quad NTF = \frac{z^2 + 1}{z^2 + z + 1}$$

zeros at $z = 0, \infty$

poles at $(-1 \pm j\sqrt{3})/2$ ← poles the same

zeros at $\pm j$

zero not on unit circle

poles $f_p = \pm f_s/3$

$\cos \omega_p T_s = -0.5$

zeros at $\pm f_s/4$

poles the same

3. $v(n) = \text{sgn}[y(n)]$

$$y(n) = y(n-1) - v(n-2)$$

$$y(0) = y(-1) - v(-2) = 0 - 1 = -1$$

$$v(0) = -1$$

$$y(1) = -1 - 1 = -2$$

$$v(1) = -1$$